ing. If decomposition of the propellants is not considered, then the use of the propellants for heat absorption is more feasible, particularly if all the propellant is used in the heat exchanger rather than only that flowing to the engine during firing. However, the chemical instability of B_2H_6, for example, at the elevated temperatures that could be attained locally or in the bulk liquid in such a heat exchanger, could present a serious problem.

Conclusions

The possibility of using heat pipes for cooling the throat area of a small rocket engine (141-lb thrust) was investigated analytically. For space operation, use of heat pipes for transfer and space radiators for rejection of the heat appears feasible. Some additional cooling must be provided, however, in order to maintain the structural characteristics of the motor case (if steel), particularly if firing times are long. Further demonstration of the technical feasibility of this concept must come from experiments and tests using actual high temperature heat sources and various heat-pipe configurations.

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Relation of Meteoroid Protection to the Luminous Efficiency

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The design of shielding to protect spacecraft from meteoroids requires a knowledge of the frequency, mass, density, and velocity of the impacting bodies. Velocity, altitude, frequency, and luminous intensity may be obtained from meteor photographs. Assumptions as to the shape and knowledge of the luminous efficiency yield the mass and density of the meteoroid. Knowledge of this mass, density, frequency, and velocity permits the spacecraft designer to select the proper thickness of meteoroid bumper for a given probability of success. However, the luminous efficiency is somewhat uncertain. In this Note the sensitivity of meteoroid protective shield thickness upon the value used for luminous efficiency is derived and is shown to be small.

The luminosity equation[1] enables one to infer the initial mass as follows:

$$I = -(\tau/2)V^2(dm/dt) \tag{1}$$

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where I = meteor luminosity, τ = luminous efficiency (exact value uncertain), V = meteor velocity, and dm/dt = ablation rate or mass loss rate.

If τ is assumed to be independent of I and V, this equation can be integrated over the path of the meteor to obtain the initial mass, m_0. This integration also yields the relationship between m_0 and τ, namely that $m_0 \propto \tau^{-1}$.

The luminous efficiency is often considered to be proportional to velocity or, i.e., $\tau = \tau_0 V$, where τ_0 is called the luminous efficiency factor. If this relation is used instead of the more basic luminosity equation

$$\int_{m_0}^{0} dm = -\left(\frac{2}{\tau_0}\right)\int_0^\infty \left(\frac{I}{V^3}\right) dt = -m_0 \tag{2}$$

and $m_0 \propto \tau_0^{-1}$ instead of $m_0 \propto \tau^{-1}$.

If the meteoroid is "spherelike," one can calculate its density from the data obtained from photographic measurements. Observations of the deceleration, velocity, and altitude when extrapolated to the initial portion of the flight can yield the parameter $m_0/C_D A_0$, where C_D = drag coefficient and A_0 = initial frontal area. Assuming the body to be a sphere of uniform density enables one to express the frontal area in terms of mass and density, $A_0 = \pi[(m_0/\rho_0)3/4\pi]^{2/3}$, where ρ_0 = initial density. Substituting this A_0 and $m_0 \propto \tau^{-1}$ into the observed value of $m_0/C_D A_0$ and solving for density yields $\rho_0 \propto \tau^{1/2}$.

Several experimenters have investigated the relationship between the meteoroid shield thickness (or penetration depth) and V, m, and ρ of the impacting projectile. These relationships, along with $m_0 \propto \tau^{-1}$ and $\rho_0 \propto \tau^{1/2}$ enable the sensitivity of the required shield thickness or penetration depth to τ to be established.

Arenz[2] correlated penetrations on double-walled 2024-T3 aluminum targets with optimum shield thickness and spacing at a fixed velocity with the expression

$$(t_s + t_b)/d_{BL} = 0.287\rho^{0.6}[1 + 0.0067m^{(0.9+0.28\rho)}] \tag{3}$$

where t_s = shield or outer wall thickness; t_b = backup sheet thickness; d = projectile diameter; subscript BL = ballistic limit, value required to resist mechanical failure on inner surface; and m and ρ for the projectile are in g and g/cm^3.

The ratio $(t_s + t_b)/d$ is only slightly dependent upon the mass term, especially for the lower density material, and the meteoroid size range of interest. The expression thus can be simplified to

$$t_s + t_b = 0.287\rho^{0.6}d \tag{4}$$

and, for a spherical shape, $d = (6m/\pi\rho)^{1/3}$. Then, with $m_0 \propto \tau^{-1}$ and $\rho \propto \tau^{1/2}$ one can write

$$(t_s + t_b)_{BL} \propto \tau^{-1/5} \tag{5}$$

In another investigation Cour-Palais[3] developed an equation for the backup sheet thickness for glass particles impacting, multiwalled 2024-T3 aluminum structures with practical values of shell spacing and front-sheet thickness. His equation has the form

$$t_b = 0.055(\rho\rho_t)^{1/6}m^{1/3}V(\text{cm}) \tag{6}$$

where t_b = backup sheet thickness, cm; ρ and ρ_t are particle and backup sheet densities, g/cm^3; and V is in km/sec. A similar analysis to that for the previous penetration equation yields $t_b \propto \tau^{-1/4}$ for a given structure and velocity.

In a third study Summers[4] demonstrates a relation for the penetration of a projectile into a solid body

$$p/d = 2.28(\rho_m/\rho_t)^{2/3}(V/C)^{2/3}$$

where C = speed of sound in the body, and p = penetration depth. Here, a similar analysis yields

$$p \propto \tau^{-1/6} \tag{7}$$

In summary, then, the relationship presented by Arenz yields $t \propto \tau^{-1/5}$ for V = 7.2 km/sec, $0.7 \leq \rho \leq 2.78$ g/cm^3,

Table 1 Comparison of shell thickness or penetration depth, meteoroid mass, and meteoroid density for various values of luminous efficiency and various penetration equations

$\bar{\tau}$	$\bar{t}$, $n = -\frac{1}{4}$	$\bar{t}$, $n = -\frac{1}{5}$	$\bar{p}$, $n = -\frac{1}{6}$	$\bar{m}$, $n = -1$	$\bar{\rho}$ $n = \frac{1}{2}$
0.1	1.78	1.58	1.47	10	0.32
1.0	1.0	1.0	1.0	1.0	1.0
10	0.56	0.63	0.68	0.1	3.2
100	0.32	0.40	0.47	0.01	10

$0.006 \leq m \leq 10.38$ g, and a multiwalled structure of 2024-T3 aluminum. Cour-Palais' relationship yields $t_b \propto \tau^{-1/4}$ for glass particles ($\rho = 2.3$ g/cm^3) for $7.5 \times 10^{-5} \leq m \leq 2.94 \times 10^{-3}$ gm impacting multilayer aluminum structures at between $5.4 \leq V \leq 6.7$ km/sec. For solid targets, Summers' relationship yields $p \propto \tau^{-1/6}$ for metal spheres with $1.5 \leq \rho \leq 17.1$ g/cm^3 fired into copper and lead targets at $0.16 \leq V \leq 3.6$ km/sec.

The uncertainties in m and ρ_m resulting from uncertainties in the value of τ are almost self-canceling when considering the meteoroid bumper penetration problem. This self-compensating effect was first noticed for single layer meteoroid shields by Whipple[6] and the same effect has been shown here to be true both for multi-wall and single-wall bumpers. Table 1 presents values for $\bar{t}$, $\bar{p}$, $\bar{m}$, and $\bar{\rho}$, which would result from variations in the values of τ over a range of three orders of magnitude. Here $\bar{\tau}$ is obtained by dividing τ by $\tau_{\rm ref}$ (an arbitrary reference value of luminous efficiency). Likewise, $\bar{m} = (m/m_{\rm ref})$, $\bar{\rho} = (\rho/\rho_{\rm ref})$, $\bar{t} = (t/t_{\rm ref})$, and $\bar{p} = (p/p_{\rm ref})$ where $m_{\rm ref}$, $\rho_{\rm ref}$, $t_{\rm ref}$, and $p_{\rm ref}$ are the m, ρ, t, and p resulting from calculations using the reference value of τ. In this table $\bar{t}$ or $\bar{p}$, $\bar{m}$ and $\bar{\rho}$ are expressed as function of τ^n.

Clearly, the meteoroid hazard is more severe than expected only if τ is significantly smaller than expected. McCrosky[7] discusses the possibility of increasing τ by a factor of 100 from an approximate value of 10^{-3} to make the luminous flux of large fireballs compatible with that from smaller meteors. This larger τ would reduce the expected meteoroid hazard from large objects by only from 53% to 68% depending upon the structure under consideration. As a practical example, Howard[5] has calculated for the Mars Mariner 71 mission a backup sheet thickness of 0.0081 cm will be required to survive an impact with a particle which has a d of 0.0792 cm, ρ of 0.5 g/cm^3, and a V of 20 km/sec. The actual shield as designed for this mission is 0.040 cm thick or 5 times as thick as required. This factor of 5 in thickness corresponds to allowing for uncertainty in the value of luminous efficiency. Using the equations relation τ and t or p in this Note, this extra shield thickness corresponds to a reduction by from 600 to 19,000 from the value of approximately 10^{-3} which is widely used for luminous efficiency.

For particles of a given V and τ, the required meteoroid bumper thickness t_s is shown to be a weak function of the luminous efficiency. Although the precise value of the luminous efficiency is uncertain, only a very large revision from its presently accepted value will significantly change the meteor bumper requirements.

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Optimization of Multistage Rockets Including Drag

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Nomenclature

weight = value at sea level
B_j = ballistic coefficient at ignition of jth stage = W_{ji}/S_j
b_j,d_j = dimensionless similarity parameters, Eqs. (26) and (27)
C_D = drag coefficient
C_j = an average exhaust gas velocity, Eq. (16)
$\bar{C}_j$ = a generalized exhaust gas velocity, Eq. (26)
D_j = dimensionless similarity parameter = $1 + R_j\bar{\beta}_j$
F = thrust
G,J = known functions of time, subscript j for burnout, Eq. (3)
$\bar{G}_j$ = similarity parameter, Table 1
g = acceleration due to gravity; g_0 at sea level
$\bar{g}_j$ = an average parameter, Eq. (16)
h_j = dimensionless similarity parameter, = $1 + R_j\bar{Q}_j$
I = specific impulse, subscript j for burnout
k_j = average motor wall thickness per unit diameter
L_m = dimensionless similarity parameter, Eq. (29)
N_j = integral of G over burning time, Eq. (5)
n = total number of stages
p = payload; $p_j = W_{(j+1)i}$; p_n = final payload = $W_{1i}\prod_{j=1}^{n}\beta_j$
Q_j = a trajectory constant, Eq. (6)
$\bar{Q}_j$ = dimensionless similarity parameter, Eqs. (19) and (20)
q = dynamic pressure
R_j = ratio of propellant to inert motor weight = $\lambda_j/(1 - \lambda_j)$
$\dot{r}_j$ = burning rate, length per second
S = reference area for drag (cross section)
T_j = burning time for jth stage = $t_{jb} - t_{ji}$
t = time
U = the dimensionless compatibility function
u_j = a generalized dimensionless velocity increment
$V,\dot{V}$ = velocity and acceleration at time t
V_t = total velocity increment = $\sum_{j=1}^{n} v_j$
v_j = velocity increment for jth stage
W = vehicle weight at time t
x_j = parameter for end-burning stoges, Eq. (18)
α = angle between thrust and velocity vectors
β_j = payload ratio of jth stage = p_j/W_{ji}; $\bar{\beta}_j$ for no drag, Eq. (10)
γ = angle between velocity vector and horizontal
λ_j = propellant mass fraction for jth rocket motor
ξ = dimensionless time function, Eq. (11)
$\bar{\xi}_j,\xi'_j$ = dimensionless similarity parameters, Table 1
ρ_j = propellant density of jth stage, Eq. (18)

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